EARNINGS AND INVESTMENT IN COMPANY VALUATION

Miller Modigliani, "Dividend policy, growth and the valuation of shares", Journal of Business, October 1961

1. Background

MM61 is the paper which first demonstrated dividend irrelevance. However, it also dealt with company valuation.

A common model before their paper was the Gordon growth model. This takes the dividend valuation model

$$V_0 = \sum_{t=0}^{t=\infty} \frac{D_t}{(1+r)^{t+1}}$$

and imposes some special assumptions. Gordon assumes that dividends grow at a constant rate, g. Therefore the dividend stream is:

<	0	><	1	><	2	><	3	><	4	> .... etc
|		|		|		|		|	
		D_0		D_1		D_2		D_3		D_4

becomes

D_0	$D_0(1+g)$	$D_0(1+g)^2$	$D_0(1+g)^3$	$D_0(1+g)^4$

Discounting and summing this becomes

$$V_0 = \frac{D_0}{(r-g)}$$

where

V_0	=	the value of the corporation at the beginning of period 0;
D_0	=	the dividends to be paid at the end of period 0;
r	=	the discount rate, the cost of capital;
g	=	the expected growth rate of dividends over the (infinite) life time of the corporation.

Following this model, there was some controversy about whether this excludes a role for earnings in the valuation of the company.

2. Contribution of MM61

MM show that a number of approaches to valuation are in fact identical. They show that a company can be valued by:

- discounting the dividend stream which would be payable to current shareholders (the dividend approach), as in the approach above;

- discounting the earnings stream, but then deducting the cost of capital on new investment (the earnings approach) - they noted that simply to discount earnings would be to double count, since shareholders do not receive earnings);

- the investment opportunities approach - it is this approach that turns out to be the most insightful with regard to the role of earnings.

Additional definitions:

X_t = cash flow arising at the end of t, from capital assets held at the beginning of t
I_t = investment undertaken at the end of t
D_t = dividends paid at the end of t, to shareholders existing at the beginning of period t

The value of the corporation is defined as the present value of future dividends, presented diagrammatically as:

	0		1		2		3		4	
	<	><		><		><		><		> .... etc
	|	|		|		|		|		|
V_0 =		D_0		D_1		D_2		D_3		D_4

However, the dividends in each period can be expressed as the difference between the cash flow received and investment made by the company. Investment here is simply cash retained by the company. It may purchase either fixed or working capital assets[1] with the cash.

	0		1		2		3		4	
	<	><		><		><		><		> .... etc
	|	|		|		|		|		|
V_0 =		X_0		X_1		X_2		X_3		X_4
		$-I_0$		$-I_1$		$-I_2$		$-I_3$		$-I_4$

The cash flow (X) in each period can be divided in to

- the cash flow which would have been received by the company if no further investment were made after the date of the company valuation, and

[1] Note that in this world, investment consists of *all* assets from current cash flow which are not paid to shareholders.

- the extra cash flow derived from investment I_0, I_1, I_2 etc (for simplicity, r* is defined as the return on all investments)

Note that this way of thinking assumes that once an investment has been made, then it generates identical cash flows in each period in perpetuity. This is strictly unrealistic, and we discuss later how it might be relaxed.

<	0	><	1	><	2	><	3	><	4	> .... etc
|		|		|		|		|		|
V_0 =										
		X_0		X_0		X_0		X_0		X_0
				$r^*.I_0$		$r^*.I_0$		$r^*.I_0$		$r^*.I_0$
						$r^*.I_1$		$r^*.I_1$		$r^*.I_1$
								$r^*.I_2$		$r^*.I_2$
										$r^*.I_3$
		$-I_0$		$-I_1$		$-I_2$		$-I_3$		$-I_4$

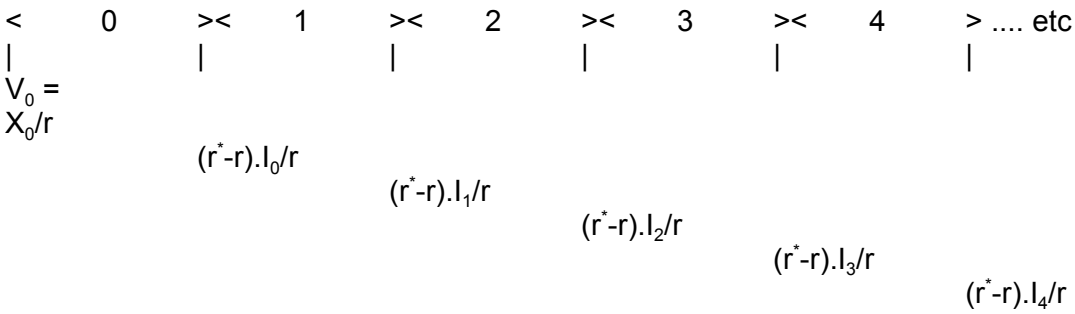
Grouping all the X_0 items together (the sum of an infinite geometric series) gives

<	0	><	1	><	2	><	3	><	4	> .... etc
|		|		|		|		|		|
V_0 =										
X_0/r										
				$r^*.I_0$		$r^*.I_0$		$r^*.I_0$		$r^*.I_0$
						$r^*.I_1$		$r^*.I_1$		$r^*.I_1$
								$r^*.I_2$		$r^*.I_2$
										$r^*.I_3$
		$-I_0$		$-I_1$		$-I_2$		$-I_3$		$-I_4$

Similarly, grouping all the I_0 items, and all the I_1 items etc. gives

<	0	><	1	><	2	><	3	><	4	> .... etc
|		|		|		|		|		|
V_0 =										
X_0/r										
		$r^*.I_0/r$								
				$r^*.I_1/r$						
						$r^*.I_2/r$				
								$r^*.I_3/r$		
										$r^*.I_4/r$
		$-I_0$		$-I_1$		$-I_2$		$-I_3$		$-I_4$

Incorporating the initial cost of the investments gives



Discounting these elements, the value of the company can be written as:

$$V_0 = \frac{X_0}{r} + \sum_{t=0}^{t=\infty} \left\{ \frac{(r^* - r).\, I_t}{r.\quad (1+r)^{t+1}} \right\}$$

The first term is the value of the cash flows from existing capital stock at the beginning of the period. This is the value of the company from its existing assets.

The second term is the excess profits from future investment opportunities to be taken up by the company. This is the value of the company from its future assets (capital stock).

3. The link with accounting numbers

I - THE FIRST TERM

$$\frac{X_0}{r}$$

This is the value of sustainable cash flow in perpetuity. Sustainable cash flows is exactly what earnings are about. They are an artificial construct of accountants using two key concepts:

Accruals — This concept requires that a transaction should be recognised in the accounts when there is an economic agreement (for example, when the incentives are clearly in favour of not returning goods which have been taken on inspection). This contrasts with other possibilities such as: when there is a legal agreement; or when cash passes to the seller.

Matching — This concept allows some costs to be deferred to a future period. This is achieved by calling some costs “assets” and amortising them over a particular period. This contrasts with charging all expenditure (including plant, equipment and buildings) to the P&L .

II - THE SECOND TERM

$$\sum_{t=0}^{t=\infty} \left\{ \frac{(r^* - r).\, I_t}{r.\,(1+r)^{t+1}} \right\}$$

Given that this information is forward looking, the annual report and accounts of the company is unlikely to be the best place for this information to be disclosed. For example, research and development disclosure is the subject of SSAP 13, but how can these opportunities be summed up by reporting past or even future expenditure !! ? How can the accounts give an estimate of r* ? Might not investors be better informed about this activity by the more subjective evaluations of say investment analysts ? This issue really boils down to the comparative advantage of disclosing information through the audited annual accounts rather than through other sources of information.

Although some accounting information is relevant to estimating this term, it is likely that information about this component is collected by information intermediaries, such as analysts.

III - IS CURRENT EARNINGS ENOUGH TO ESTIMATE X_0?

Consider the variable X_0 we need.

<	-1	><	0	><	1	><	2	><	3	> .... etc	
|		|		|		|		|		
		V_0 =									
				X_0		X_0		X_0		X_0	

The variable X_0 we need for V_0 is the sustainable cash flow from the existing capital stock which is in place at the beginning of period 0. In fact if we are trying to value the company at the beginning of period 0, this is next year's earnings. But the latest earnings information we will be X_{-1}; and this will have been generated by the capital stock at the beginning of period -1, K_{-1}.

<	-1	><	0	><	1	><	2	><	3	> .... etc	
K_{-1}		X_{-1}									
		K_0		X_0							

Therefore an estimate of X_0 should be constructed, assuming that the expected rate of return on capital remains the same as before,

$$\hat{X}_0 = K_0 . \hat{r}$$

where

$$\hat{r} = \frac{X_{-1}}{K_{-1}}$$

Therefore

$$\hat{X}_0 = X_{-1} \frac{K_0}{K_{-1}}$$

4. A more realistic model

One of the simplifications of MM61 is that it does not deal with uncertainty. However, one of the advantages of model building is to isolate problems, making some parts of the model complicated and some parts of it simple.

A more worrying assumption is that of perpetuities. When a company makes an investment, cash does not flow in perpetuity. One way in which the model can be made more realistic is to assume that cash flows are in constant decline.

existing capital stock generates X_0, but that this declines over time at rate d, as follows: X_0, $X_0(1 + d)$, $X_0(1 + d)^2$

investment can then be reasonably defined as a constant proportion (z), of current earnings, that is:
$I_0 = z. X_0$
$I_1 = z. X_1$

and that the return from these investments also declines over time at rate d.

Given these assumptions and definitions the value of the company can be specified as:

$$V_0 = \frac{X_0}{r - d} + \frac{z.X_0(r^* - r + d)}{(r - d)(r - d - zr^*)}$$

5. A simpler model

Sometimes a rough and ready model is all that is needed. It may not be worth the effort of estimating the investment to be made in each period. If a simpler approach is warranted, then the MM formula can be adjusted. We can write a model in which many of the cash flows are expressed as perpetuities. This might be called a super perpetuity model. Specifically:

1 the sustainable cash flow from existing assets is constant for all periods ;
2 the amount of investment, and its return remain constant for all periods;
3 the sustainable cash flow from investment (new assets) is constant for all periods.

The basic MM model is

$$V_0 = \frac{X_0}{r} + \sum_{t=0}^{t=\infty} \left\{ \frac{(r^* - r).I_t}{r.\ (1+r)^{t+1}} \right\}$$

which already incorporates assumptions 1 and 3 above. If we define further that

I_t = I for all t, then the MM equation becomes

$$V_0 = \frac{X_0}{r} + \frac{I.(r^* - r)}{r^2}$$

6. The free cash flow model

Quite a popular approach model in practice is the free cash flow model. Typically, the expositions of the model is strong on the mechanics used to calculate the estimated value but rather weak on rationale. In fact the approach is very similar to MM61.

Recall that in the MM61approach, dividends are defined as the difference between cash flow and investment

$$D_t \equiv X_t - I_t \qquad \text{for all t}$$

In order to make this operational, X_t is then divided in to (i) the cash flow which is generated from assets already in place at the time that the valuation is made and (ii) the extra cash flow generated from investment at the valuation date. We have suggested that the item (i) is in fact very close to accounting earnings at the valuation date. This enables the whole stream of dividends over time (D_t t=0,1,2,3,4 .....) to be estimated by a single earnings number and information about future investments.

It should also be noted that since we use earnings to estimate the sustainable cash flow component, it is right that this estimate is *net of depreciation* because equipment has to be replaced as it wears out. Earnings has this characteristic. This means that item (ii) is the extra cash flow from *net* investment (that is, after replacement investment has been taken in to account).

In contrast to this, the free cash flow approach to valuation seems to ignore the accounting concepts of depreciation and accruals, and estimate the future dividend stream directly, as follows.

$$\begin{aligned} D_t &\equiv X_t - I_t \\ &\equiv \text{Operating cash flow} - \text{Gross investment} \\ &\equiv \text{Free cash flow} \end{aligned}$$

X_t is interpreted to mean cash flow without the expense of replacement investment, and I_t is then interpreted to be all investment (for replacement and expansion). Free cash flow is then estimated for each future period after the valuation date.